CCFU Proof 29

C_2 Constructs G_2(split)

Theorem. C_2: $x(n+1) = x(n) + x(n-1)$ determines a stable 3-form Ω_W on $\mathbb{R}^7$, unique up to GL(7)-equivalence, whose stabilizer is G_2(split). One volume choice on W, absorbed by GL(7).

Proof.

(i) C_2 forces $\varphi = (1+\sqrt{5})/2$ [Proof 1].

(ii) $\mathrm{spec}(A_2) = \{\varphi, -1/\varphi\}$ gives two completions: factor $\to$ sig$(3,1)$, symmetry $\to$ sig$(2,2)$ [Proof 17].

(iii) sig$(2,2)$ confirmed universal at fixed point for all degree-2 memory maps [Proof 26].

(iv) Minimal parent: sig$(3,2)$, $\dim 5 = \Delta(C_2)$ [Proofs 5, 17].

(v) Parent decomposes: $W = S_1 \cap S_2$, $\dim W = 3$. W nondegenerate, sig$(2,1)$ [Proof 20].

(vi) Higher alternating closure [Proof 21]:
$V_W = \mathbb{R} \oplus W \oplus W^*$, $\dim = 1 + 3 + 3 = 7$.
$\Omega_W = \tau \wedge \mathrm{ev} + \mathrm{vol}_W + \mathrm{vol}_{W^*}$.
Five terms. Volume choice on W absorbed by GL(7).
dim 7 is minimal.

(vii) $\dim \mathrm{Stab}(\Omega_W) = 14$, exact [Proof 23].

(viii) $\mathrm{sig}(b_{\Omega_W}) = (3,4)$, algebraic [Proof 24].

(ix) Ω_W stable: $\dim \mathrm{orbit} = 35 = \dim \Lambda^3$ [Proof 25].

(x) GL(7)-orbit unique: all $\Omega_{a,b,c}$ with $abc \neq 0$ equivalent [Proof 27].

(xi) $\mathrm{Stab}(\Omega_W) \cong \mathfrak{g}_2$(split) [Proof 28]. ■

Uniqueness.
C_2 is unique among ± 1 recurrences (Proof 18).
W is forced by signature constraints (Proof 20).
$V_W = \mathbb{R} \oplus W \oplus W^*$ is the canonical higher closure (Proof 21).
Ω_W is the unique natural 3-form up to GL(7) (Proof 21).
One volume choice on W. It does not affect the GL(7)-orbit.

[Dependencies: Proofs 1, 5, 17, 18, 20–28.
External: Killing 1887, Cartan 1914, Cartan criterion, Schur.]